The Energy Equation for Two-Phase Flow

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In a communication recently published in the *A.I.Ch.E. Journal* (1) H. S. Isbin and Yung Sung Su considered the problem of the proper energy equation for a two-phase flow system. They wrote the energy equation per pound mass for a single-phase, constant density, time independent, flow system as

$$1/\rho \frac{dP}{dz} + \frac{VdV}{g_c dz} + \frac{g}{g_c} + \frac{dF}{dz} = 0 \quad (1)$$

They noted that Equation (1) is applicable to a two-phase system only if no mass transfer occurs between phases. They then contended that the proper way to apply Equation (1) to a two-phase flow system is to weight the energy equation per pound mass of each phase by the mass of the phase contained in the differential section dz ($\rho_G A_G\, dz$ for the gas phase and $\rho_L A_L\, dz$ for the liquid phase) and then sum the equations.

Other investigators however have applied Equation (1) to a two-phase flow system by weighting the energy equation for each phase by the appropriate mass flow rate (W_G or W_L) and then adding the equations.

In order to see which of these weighting procedures leads to the correct expression for the energy equation of a two-phase flow, consider a differential section dz of the vertical two-phase flow shown in Figure 1.

For constant density and constant mass flow rate of each phase the energy equation per pound mass of the gas phase is

$$\frac{1}{\rho_G}\frac{dP}{dz} + \frac{V_G}{g_c}\frac{dV_G}{dz} + \frac{g}{g_c} + \frac{du_G}{dz} + \frac{1}{W_g}\frac{dw}{dz} + \frac{1}{W_G}\frac{dq}{dz} = 0 \quad (2)$$

and per pound mass of the liquid phase is

$$\frac{1}{\rho_L}\frac{dP}{dz} + \frac{V_L}{g_c}\frac{dV_L}{dz} + \frac{g}{g_c} + \frac{du_L}{dz} - \frac{1}{W_L}\frac{dw}{dz} - \frac{1}{W_L}\frac{dq}{dz} = 0 \quad (3)$$

In Equations (2) and (3) dw represents the rate at which shear work is done by the gas phase on the liquid phase across the interface in the section dz, and dq represents the rate at which heat is transferred from the gas phase to the liquid phase across the interface in the section dz. From Equations (2) and (3) it is seen that in applying Equation (1) to either phase of a two-phase flow the term dF/dz must be recognized as including changes in internal energy, work done between phases, and heat transferred between phases. That is

$$\frac{dF_G}{dz} = \frac{du_G}{dz} + \frac{1}{W_G}\frac{dw}{dz} + \frac{1}{W_G}\frac{dq}{dz} \quad (4)$$

INFORMATION RETRIEVAL*

Key Words: Spraying-I, Atomizing-I, Dispersing-I, Flashing-I, Aerosols-I, Jets-I, Liquids-H, Suspensions-H, Mists-H, Sprays-H, Dispersions-H, Pressure-H, Water-A, "Freon"® Fluorinated Hydrocarbons-A, Halogenated Hydrocarbons-A, Temperature-F, Heating-F, Superheating-F, Drops (Droplets)-G, Size-G, Velocity-G, Patterns-G, Orifices-J, Atomizers-J.

Abstract: Liquids forced from a high pressure zone into a low pressure zone often cross the equilibrium pressure for the liquid temperature and disintegrate into a spray by partial evolution of vapor. A study has been made of the sprays formed by such a process and of the mechanism of spray formation. Sprays from water and Freon-11 jets were analyzed for drop size, drop velocities, and spray patterns. The break-up mechanism was analyzed and data are presented to show some of the controlling factors. The effects of temperature, Weber number, and type of orifice are considered.

Reference: Brown, Ralph, and J. Louis York, **A.I.Ch.E. Journal 8,** No. 2, p. 149 (May, 1962).

Key Words: Rheology-H, Heat Transfer-H, Flow-H, Fluid Flow-H, Laminar Flow-H, Fluids-I, Non-Newtonian-, Viscosity-I, Laminar Flow-I, Tubes-J, Temperature-F, Rates-G, Transport-G, Conduction-G, Numbers-I, Nusselt-, Graetz-, Predictions-I, Models-J.

Abstract: A simple temperature dependent equation is proposed to represent the rheological properties of pseudoplastic fluids. Combining the equation with the differential equations for steady state laminar flow and heat conduction in circular tubes with constant wall temperature, numerical solutions are obtained for both Newtonian and non-Newtonian fluids. The calculated results are presented graphically, and are compared with experimental data for two pseudoplastic fluids.

Reference: Christiansen, E. B., and S. E. Craig, Jr., **A.I.Ch.E. Journal, 8,** No. 2, p. 154 (May, 1962).

Key Words: Equilibrium-I, Thermodynamics-I, Predicting-I, Empirical-I, Methods-I, Techniques-I, Phases-H, Vapors-H, Liquids-H, Fluids-H, Binary-H, Hydrocarbons-H, Water-H, Pressure-F, Equilibrium-G.

Abstract: This paper presents an empirical method for the prediction of vapor-liquid equilibrium data of a binary system at various pressures on the basis of equilibrium data near atmospheric pressure. The proposed method is tested for six hydrocarbon systems and one aqueous system. The calculated results are in good agreement with the experimental data.

Reference: Tamura, Mikio, and Isamu Nagata, **A.I.Ch.E. Journal, 8,** No. 2, p. 161 (May, 1962).

Key Words: Equilibrium-I, Thermodynamics-I, Predicting-I, Empirical-I, Methods-I, Techniques-I, Phases-H, Vapors-I, Liquids-H, Fluids-H, Binary-H, Hydrocarbons-H, Pressure-F, Temperature-F, Equilibrium-G.

Abstract: This article proposes an empirical method for predicting the vapor-liquid equilibrium data on binary hydrocarbon systems without any experimental data on the mixtures. The method was tested for 26 systems under atmospheric pressure and three isothermal systems. The predicted results are in excellent agreement with experimental data.

Reference: Tamura, Mikio, and Isamu Nagata, **A.I.Ch.E. Journal, 8,** No. 2, p. 163 (May, 1962).

* For details on the use of these key words and the A.I.Ch.E. Information Retrieval Program, see **Chem. Eng. Progr., 57,** No. 5, p. 55 (May, 1961), No. 6, p. 73 (June, 1961).

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$$\frac{dF_L}{dz} = \frac{du_L}{dz} - \frac{1}{W_L}\frac{dw}{dz} - \frac{1}{W_L}\frac{dq}{dz}$$

For flow in a duct with no heat transfer into the duct and no shaft work done by the flow, conservation of energy leads to the well-known result that the integral in Equation (5)

$$\int_A \rho V\left(\frac{P}{\rho} + u + \frac{V^2}{2g_c} + \frac{g}{g_c}z\right) dA = \text{const} \quad (5)$$

taken over any duct cross section normal to the flow is a constant. Applying Equation (5) directly to the total two-phase flow in the section dz one obtains Equation (6):

$$\frac{d}{dz}\left[\rho_G V_G A_G\left(\frac{P}{\rho_G} + u_G + \frac{V^2_G}{2g_c} + \frac{g}{g_c}z\right) + \rho_L V_L A_L\left(\frac{P}{\rho_L} + u_L + \frac{V^2_L}{2g_c} + \frac{g}{g_c}z\right)\right] = 0 \quad (6)$$

Performing the differentiation, rearranging terms, and dividing through by $V_G A_G + V_L A_L$ one obtains Equation (7):

$$\frac{dP}{dz} + \frac{1}{V_G A_G + V_L A_L}\left[W_G\frac{V_G}{g_c}\frac{dV_G}{dz} + W_L\frac{V_L}{g_c}\frac{dV_L}{dz} + \frac{g}{g_c}(W_G + W_L) + W_G\frac{du_G}{dz} + W_L\frac{du_L}{dz}\right] = 0 \quad (7)$$

This is the correct energy equation for the combined two-phase flow.

Now consider Equations (2) and (3). If Equation (2) is weighted by W_G and Equation (3) is weighted by W_L and the two equations are added, the terms representing the heat transfer

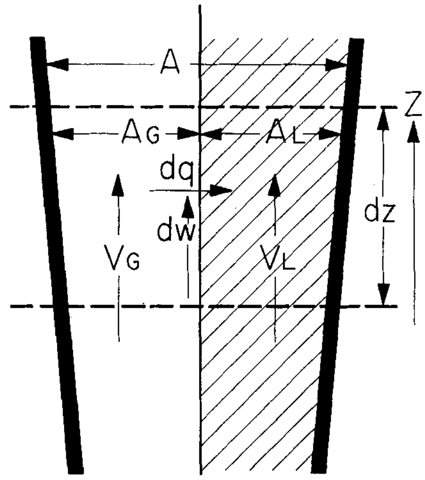

Fig. 1.

and work done between phases cancel out and Equation (7), the correct two-phase flow energy equation is obtained. If however one weights Equation (2) by $\rho_G A_G dz$ and Equation (3) by $\rho_L A_L dz$ and sums, the terms representing the heat transfer and work done between phases do not cancel out and one obtains an equation which, unlike Equation (7), does not have a clear physical interpretation as the energy equation for the total flow.

An objection raised by the authors of reference *1* to the use of W_G and W_L as weighting factors is that in dt time the gas phase moves a distance $W_G\, dt/\rho_G A_G = V_G\, dt$, whereas the liquid moves $W_L\, dt/\rho_L A_L = V_L\, dt$. Thus the only way for the incremental distance dz to be the same for the gas and liquid phase is for $V_G = V_L$.

Since it is not precisely clear to this present writer how this objection is intended to apply to the differential equations presented here, no attempt will be made to discuss the objection However in view of the demonstrated correctness of Equation (7) as the energy equation for a total two-phase flow, the objection raised by Isbin and Yung Sung Su to the use of W_G and W_L as weighting factors would seem to be not valid.

NOTATION

A = total cross-sectional area of duct, sq. ft.
A_G = cross-sectional area of flow for gas phase, sq. ft.
A_L = cross-sectional area of flow for liquid phase, sq. ft.
$(dF)/(dz)$ = derivative representing rate of change of fluid internal energy plus rate of change of flow energy due to heat transfer and shear work, ft.-lb./lb. mass-ft.
g = gravitational acceleration, ft./hr.^{2}
g_c = gravitational constant, 4.17×10^8 ft./hr.^{2}
P = pressure, lb./sq. ft.
t = time, hr.
V = velocity, ft./hr.
W = mass flow rate, lb. mass/hr.
z = vertical height, ft.
ρ = density, lb. mass/cu. ft.
u = thermodynamic internal energy, ft.-lb./lb.mass
dw = incremental rate of work done by gas phase or liquid phase in length dz, ft.-lb./hr.
dq = incremental rate of heat transfer to liquid phase from gas phase in length dz, ft.-lb./hr.

Subscripts

G = gas phase
L = liquid phase

LITERATURE CITED

1. Isbin, H. S., and Yung Sung Su, *A.I.Ch.E. Journal*, 7, 174 (1961).

Use of Momentum and Energy Equations in Two-Phase Flow

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Recently some discussion has arisen as to the proper use of the momentum equation and mechanical energy equation when applied to two-phase flow systems (*5*). A number of publications have appeared in which pressure drop in two-phase fluid flow has been correlated in terms of the total wall shearing stress, while in other work the energy dissipation has been used as the basis of correlation. The purpose of this communication is to present derivations of momentum and mechanical energy equations and to show their relation to pressure-drop correlations in two-phase flow.

Consider first a single fluid phase. The momentum equation at a point can be written as

$$\frac{1}{g_c}\left[\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla\cdot(\rho\mathbf{V}\mathbf{V})\right] = -\nabla p + \nabla\cdot\overline{\overline{\tau}} + \rho\mathbf{F} \quad (1)$$

The mechanical energy equation is obtained from the momentum equation by forming a scalar product with the velocity vector (*8, 3*):

$$\frac{1}{2g_c}\left[\frac{\partial(\rho V^2)}{\partial t} + \nabla\cdot(\rho V^2\mathbf{V})\right] = -\mathbf{V}\cdot\nabla p + \nabla\cdot(\overline{\overline{\tau}}\cdot\mathbf{V}) - (\overline{\overline{\tau}}\cdot\nabla)\cdot\mathbf{V} + \rho\mathbf{F}\cdot\mathbf{V} \quad (2)$$

For one-dimensional, incompressible, steady flow of a gas Equation (1) can